

Quantum Optics

D.F. Walls Gerard J. Milburn

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D.F. Walls, F.R.S.
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Gerard J. Milburn
Physics Department
University of Queensland
St Lucia, Brisbane QLD 4072, Australia
milburn@physics.uq.edu.au

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Preface to Second Edition

The field of quantum optics today is very different from the field that Dan Walls and I surveyed in 1994 for the first Edition of this book. Some of the new fields that have emerged over the years were hinted at in the earlier edition: quantum information has at least some roots in the study of Bell's Inequalities, while the fields of ion trapping and quantum condensed gases have their roots in the old chapter on light forces. However such is the growth of activity in each of these areas that I have found it necessary to write four new chapters for this edition. In order to keep the book to a reasonable size this has meant cutting some of the material from the first edition. The old chapter on Intracavity Atomic Systems is largely gone with parts distributed in the new chapter on Cavity QED and elsewhere. Likewise the old chapter on Resonance Fluorescence has been redistributed across Chaps. 10 and 11 in this edition. No doubt more cutting could have been made but I have tried to keep some continuity with the previous edition. In any case an emphasis on experimental realisations has been retained in the new material. Preparing this edition was not as much fun as the first. With Dan Walls untimely death in 1999, I have been denied the consolations of a shared task and soldiered on alone (although I must admit to hearing his voice from time to time as I cut and pasted). I can only hope that I have not lost his vision for the book in my unchallenged role of sole author.

Brisbane, Australia,
October 2007.

G.J. Milburn

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